Ph136a. Lecture 1. Sept 26, 2011.

- Course description etc.

- No ~~courses~~ classes Oct 10 & 12. either make up ~~...~~ or have to skip...

- Structure of this course. One topic each week

  - Geometric description of physics, special relativity.

  - Statistical Mechanics. General Ideas & key concepts. Not as detailed or deep as ph 127. Stochastic Process

  - Optics. Interference. Diffraction. Holography. non linear optics

  - Elasticity.

  - Fluid Dynamics

  - Plasma physics

  - General Relativity & Cosmology

$$
\begin{tikzcd}
\text{"Quantum Gravity"} \arrow[d, "\text{classical gravity}"] \arrow[dr, bend left, "\text{classical}"] & \\
\text{QFT on curved spacetime} \arrow[r, "\text{classical}"] \arrow[d, "\text{no gravity}"] & \text{General Relativity.} \arrow[d, "\text{no gravity.}"] \arrow[ddd, bend right=60, "\text{"weak field" "low speed"}"'] \\
\text{QFT on flat background} \arrow[r, "\text{classical}"] \arrow[d, "\text{low speed}"] & \text{Special Relativity} \arrow[d, "\text{low speed, low stress}"] \\
\text{NR QM} \arrow[r, "\text{classical}"] \arrow[dr, leftrightarrow, "??"'] & \text{Newtonian physics (no gravity)} \arrow[d, "\text{add gravity}"] \\
 & \text{Newtonian Gravity}
\end{tikzcd}
$$

• Newtonian physics from ~~in the~~ "Geometric Point of view"

(I) Motivation

unique, simple — physics in terms of vectors, tensors

$$\Big\downarrow \text{ choice of coordinate system}$$

. complicated, dependent on representation, observer, etc. — relations among "components" or coordinate representations.

• $\vec{L} = \overleftrightarrow{I} \cdot \vec{\Omega}$, or $\underset{\text{vector}}{\underline{L}} = \underset{\text{tensor}}{\mathbb{I}}(\underset{\text{vector}}{\underline{\Omega}})$

$L_j = I_{jk}\,\Omega_k$ ← component relations

• $\underline{\nabla} \cdot \underline{A} \rightarrow \partial_k A_k$

cylindrical

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_r) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

use this to formulate physical laws.

$$\partial_r A_r + \ldots A_\theta + \ldots A_\phi$$

$$\partial_r A_r + \frac{1}{r}\partial_\phi A_\phi + \partial_z A_z$$

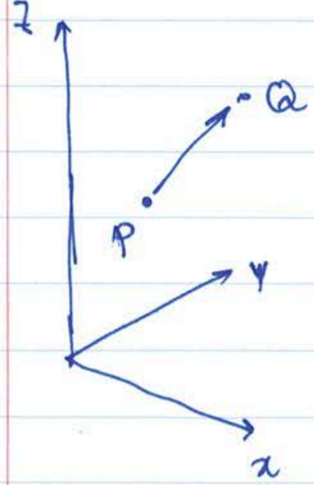
$$\ldots$$

use these to compute for convenience depending on situation

Euclidean → Minkowski → General

(II) Euclidean Space. Vectors.

A. vectors



i) scalar & scalar field.

ii) vector. P to Q

choose O, $\vec{OP}$. $\vec{OQ}$

$$\underline{x}_P = \underline{OP}, \quad \underline{x}_Q = \underline{OQ}$$

o Length $|\Delta \underline{x}|^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$

this is "invariant". independent of coordinate systems used.

• Example vector $P(\lambda)$, then

$$\underline{v} = dP/d\lambda = d\underline{x}/d\lambda$$

is a vector field.

• A Geometrical Law. $\underline{F} = m\underline{a} = m\, d\underline{v}/dt$

$$\underline{v} = d\underline{x}/dt$$

B. Tensors tensor is a multilinear function

\# of slots = rank of tensor

1. $$\mathbb{T}(\underbrace{-,-,\ldots,-}_{n \text{ slots}})$$

$$\mathbb{T}(e\underline{E} + f\underline{F}, \underline{B}, \underline{C}) = e\mathbb{T}(\underline{E}, \underline{B}, \underline{C}) + f\mathbb{T}(\underline{F}, \underline{B}, \underline{C})$$

2. metric tensor

length is geometrical

$$\underline{A} \cdot \underline{B} \equiv \frac{1}{4}\left[(\underline{A}+\underline{B})^2 - (\underline{A}-\underline{B})^2\right]$$

length is geometrical.

geometrical quantity.

$$g(\underline{A}, \underline{B}) \equiv \underline{A} \cdot \underline{B}$$

or $$g(\_, \_) = \_ \cdot \_$$

$$g(\underline{A}, \underline{B}) = g(\underline{B}, \underline{A})$$

3. vector as tensor (special for Euclidean)

for $\underline{A}$, $\mathbb{A}(\underline{C}) \equiv \underline{A} \cdot \underline{C}$ from vector to rank-1 tensor

$\boxed{\text{for } \mathbb{A}(\_).\ \exists\ \underline{A}.\ \text{s.t. } \forall \underline{C},\ \mathbb{A}(\underline{C}) = \underline{A} \cdot \underline{C}}$ needs proof

$$\underline{J} : \quad \mathbb{I}(\_, \_)$$

$$\downarrow$$

$$\mathbb{J}(\_) = \mathbb{I}(\_, \underline{\Omega})$$

symmetric anyway.

4. outer products.

$$\mathbb{A} \otimes \mathbb{B} \otimes \mathbb{C} (\underline{E}, \underline{F}, \underline{G}) = \mathbb{A}(\underline{E})\, \mathbb{B}(\underline{F})\, \mathbb{C}(\underline{G})$$

$$\underset{\substack{\uparrow \\ 2}}{\mathbb{T}} \otimes \underset{\substack{\uparrow \\ 3}}{\mathbb{S}} (\underline{E}, \underline{F}, \underline{G}, \underline{H}, \underline{J}) = \mathbb{T}(\underline{E}, \underline{F})\, \mathbb{S}(\underline{G}, \underline{H}, \underline{J})$$

5 contraction

$$\text{Contraction } (\mathbb{A} \otimes \mathbb{B}) = \underline{A} \cdot \underline{B}$$

C. components of tensors

1. Vectors $\underline{A} = A_j \underline{e}_j$

basis vectors $j = 1, 2, 3$

components

2. Any tensor (to be proven)

(i) $\mathbb{T} = T_{ijk} \underbrace{\underline{e}_i \underline{e}_j \underline{e}_k}$

components

basis for rank 3 tensors

(ii) $T_{ijk} = \mathbb{T}(\underline{e}_i, \underline{e}_j, \underline{e}_k)$

$\Rightarrow \; g_{ij} = \underline{e}_i \cdot \underline{e}_j = \delta_{ij}$

apply contraction to each component

3. contraction $\mathbb{T}(\_, \_, \_) \rightarrow \quad T_{iji}\, \underline{e}_j$

contraction

D. Slot - Naming Index Notation

1. $F(\_,\_) \rightarrow F(\_a, \_b) \rightarrow F_{ab}$.

2. manipulation same as if $a: 1 \rightarrow 3$. $b: 1 \rightarrow 3$.

E. Basis orthonormal basis.

1. $\underline{e}_i = \underline{e}_{\bar{p}} R_{\bar{p}i}$, $\underline{e}_{\bar{p}} = \underline{e}_i R_{i\bar{p}}$

~~orthogon~~

~~(II) Other Tensors~~

both must be orthogonal matrices...

$$\delta_{ij} = \underline{e}_i \cdot \underline{e}_j = R_{\bar{p}i} R_{\bar{q}j} \delta_{\bar{p}\bar{q}}$$

$$\|R\| \, \|R^T\| = \| I \|$$

2. Rotation Group. $\det \|R\| = 1$.

Derivative.

(II) Other Tensors

A. derivative $\mathbb{T}(P)$: tensor field.

$$\nabla_{\underline{A}} \mathbb{T} = \lim_{\epsilon \to 0} \left[\mathbb{T}(\underline{x}_P + \epsilon \underline{A}) - \mathbb{T}(\underline{x}_P) \right] / \epsilon$$

$$\nabla_{\underline{A}} T = \underline{\nabla} T(\_, \dots, \_, \underline{A}) \rightarrow T_{abc;d} A_d$$

in Cartesian coordinate system

$$T_{ijk;l} = \frac{\partial T_{ijk}}{\partial x_l} = T_{ijk,l}$$

because we have the same basis everywhere!!

will be complicated when basis changes over location.

2. $\underline{\nabla} \cdot A = A_{j;j}$

3. $\nabla^2 \mathbb{T} = (\underline{\nabla} \cdot \underline{\nabla}) \mathbb{T} = T_{abc;jj}$

B. Levi-Civita Tensor

1. totally antisymmetric over 3 components
2. $\epsilon(\underline{e}_1, \underline{e}_2, \underline{e}_3) = +1$ for orthonormal basis ~~and right-handed~~

prove this is preserved for orthonormal right-handed basis

3. $\underline{A} \times \underline{B} = \epsilon(\_, \underline{A}, \underline{B})$

$(\underline{A} \times \underline{B})_i = \epsilon_{ijk} A_j B_k$

simplifies identities involving "×"

$(\underline{\nabla} \times \underline{A})_i = \epsilon_{ijk} A_{k;j}$

4. $\epsilon_{ijk} \epsilon_{ilm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

(III) Volumes & conservation Laws.

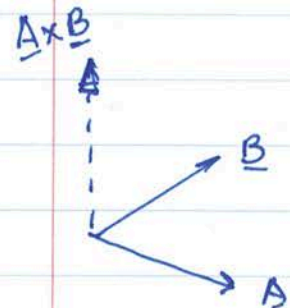
A. 1. 2-D volume

$$\epsilon_{ab} A_a B_b = \det \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$$

~~B~~. 2. 3-D. volume

$$\epsilon_{ijk} A_i B_j C_k = \underline{A} \cdot (\underline{B} \times \underline{C}) = \det \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

~~B~~. 3-D oriented area



$$\underline{\Sigma} = \underline{A} \times \underline{B} = \epsilon(\_, \underline{A}, \underline{B})$$

$$\text{Volume} = \underline{\Sigma}(\underline{C}) = \epsilon(\underline{C}, \underline{A}, \underline{B})$$

B. Conservation law

Levi-Civita gives volume
conservation Law has
~~macro~~ integral &
differential
forms.

$$\frac{\partial}{dt} \int_{V_3} \rho_e \, dV + \int_{\partial V_3} \underline{j} \cdot d^2\underline{\Sigma} = 0$$

via Gauss Theorem

$$\downarrow$$

$$\int_{V_3} \frac{\partial \rho}{\partial t} dV + \int_{V_3} \underline{\nabla} \cdot \underline{j} \, d^3\Sigma = 0 \Rightarrow \partial_t \rho + \underline{\nabla} \cdot \underline{j} = 0$$

requires no coordinate
system

(IV) The stress tensor.

surface area

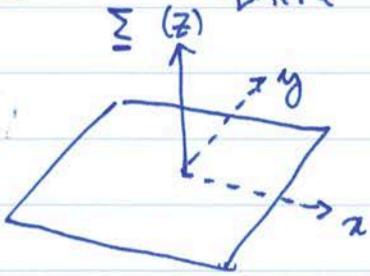
A. $F(-) = \mathbb{T}(-, \underline{\Sigma})$

vector

stress tensor

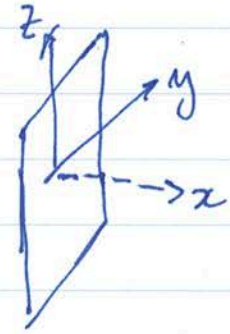
momentum / time · Area

B. there $\mathbb{T}$: "force per unit area..."



F_{zz} : force along z, exerted across x-y plane

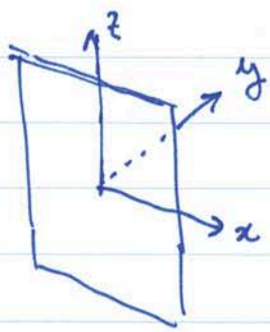
pressure!



F_{yx} force along y

F_{zx} force along z

} exerted across y-z plane



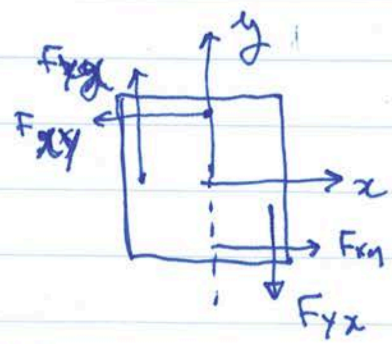
F_{xy}

F_{zy}

} shear force...

F_{yy} → pressure

C. Symmetry



$F_{xy} - F_{yx}$

cube

torque $\left(F_{xy} - F_{yx} + F_{xy} - F_{yx}\right) L^2 \cdot L/2$

moment of inertia $\sim \rho L^3 \cdot L^2 \sim \rho L^5$

$d\Omega_z/dt \sim 1/L^2 \rightarrow +\infty$ unless $F_{xy} = F_{yx}$

D. Example 1. perfect fluid.

$$\mathbb{T} = p\, g$$

2. EM field. Maxwell Stress

$$T = \frac{1}{8\pi}\left[\left(\underline{E}^2 + \underline{B}^2\right) g - 2\left(\underline{E}\otimes\underline{E} + \underline{B}\otimes\underline{B}\right)\right]$$

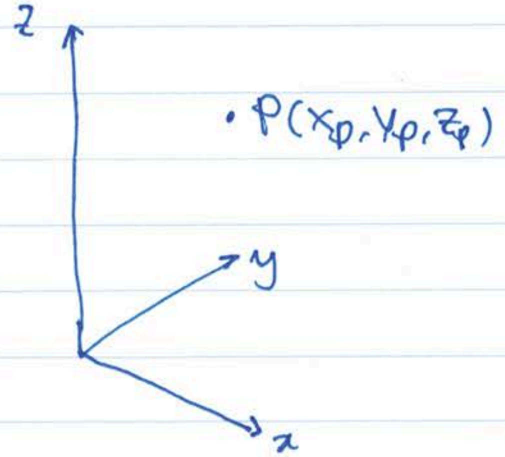
F. Conservation of momentum

$$\boxed{\partial \rho_{\underline{p}}/\partial t + \nabla \cdot \mathbb{T} = 0}$$

Week 1

Ph 136 Lecture 2 . 2011

Last time



- Euclidean Space. Vectors
- Tensors as functions of vectors ↑ linear
- metric tensor
  $g(\underline{e}_i, \underline{e}_j) = \delta_{ij}$
- Change of basis
  in particular orthogonal matrices
- Slot-Naming Notation

(II) More tensors. $\underline{\nabla}$, ϵ

A. Derivative.

$$\nabla_{\underline{A}} \mathbf{T} = \lim_{\epsilon \to 0} \left[\mathbf{T}(\underline{x}_p + \epsilon \underline{A}) - \mathbf{T}(\underline{x}_p) \right] / \epsilon$$

$$\mathbf{T} = T_{ijk}(\underline{x}) \underbrace{e_i e_j e_k}_{\text{globaly the same}}$$

$$\nabla_{\underline{A}} \mathbf{T} = T_{ijk,l} A_l \equiv \underline{\nabla} \mathbf{T}(-,-,-, \underline{A})$$

$$\equiv \underbrace{T_{ijk;l} A_l}_{\text{tensor}}$$

components → $\boxed{\dfrac{\partial T_{ijk}}{\partial x_l}}$

from $\underline{\nabla}$ define $\underline{\nabla}\cdot\underline{A} \equiv A_{j;j}$

$$\underline{\nabla}^2 T = (\underline{\nabla}\cdot\underline{\nabla})\, T = T_{abc;jj}$$

B. Levi-Civita Tensor.

1. totaly anti-symmetric rank-3 tensor
   fixed up to constant

2. define $\epsilon(e_1, e_2, e_3) = +1.$

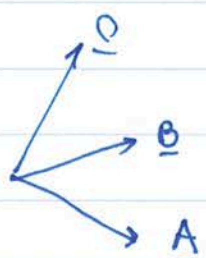
   same for right-handed orthonormal bases

3. $\underline{A}\times\underline{B} \equiv \epsilon(-, \underline{A}, \underline{B})$

$$(\underline{A}\times\underline{B})_i = \epsilon_{ijk} A_j B_k$$

$$(\underline{\nabla}\times\underline{A})_i = \epsilon_{ijk} A_{k;j}$$

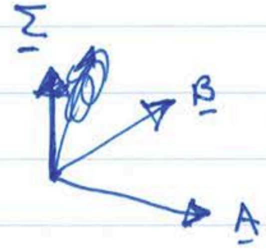
4. Volume.



$$V = \epsilon(\underline{A}, \underline{B}, \underline{C}) = \underline{A}\cdot(\underline{B}\times\underline{C})$$

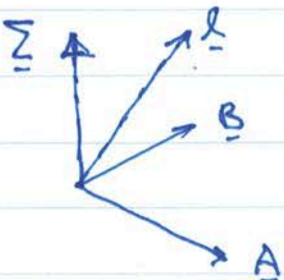
$$= \det \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

5. Area.



$$\underline{\Sigma} = \epsilon(\_, \underline{A}, \underline{B})$$

$$\underline{\Sigma} = \underline{A} \times \underline{B}, \quad \Sigma_i = \epsilon_{ijk} B_j A_k$$



$$\boxed{V = \underline{\ell} \cdot \underline{\Sigma} = \epsilon(\underline{\ell}, \underline{A}, \underline{B})}$$

6. conservation law.

$$\frac{d}{dt} \int_{V_3} \rho \, dV + \int_{\partial V_3} \underline{j} \cdot d^2 \underline{\Sigma} = 0$$

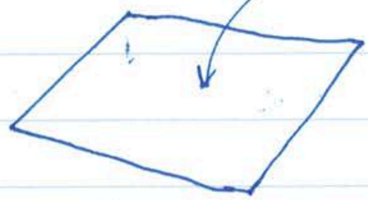
$\downarrow$ Gauss Theorem

$$\int_{V_3} (\underline{\nabla} \cdot \underline{j}) \, d^3 \Sigma$$

$$\Rightarrow \partial \rho / \partial t + \underline{\nabla} \cdot \underline{j} = 0$$

(II) Stress Tensor

A. force exerted across the surface.

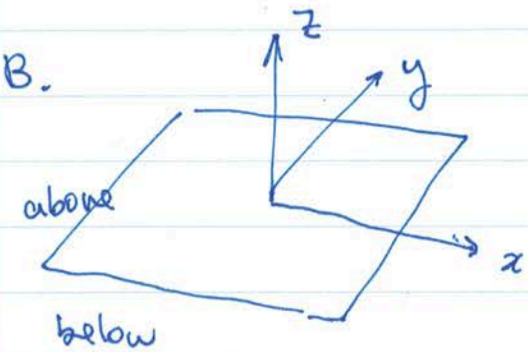


$$\mathbb{T}(\_) = \mathbb{T}(\_, \underline{\Sigma})$$

stress tensor — area

force per unit area.

B.



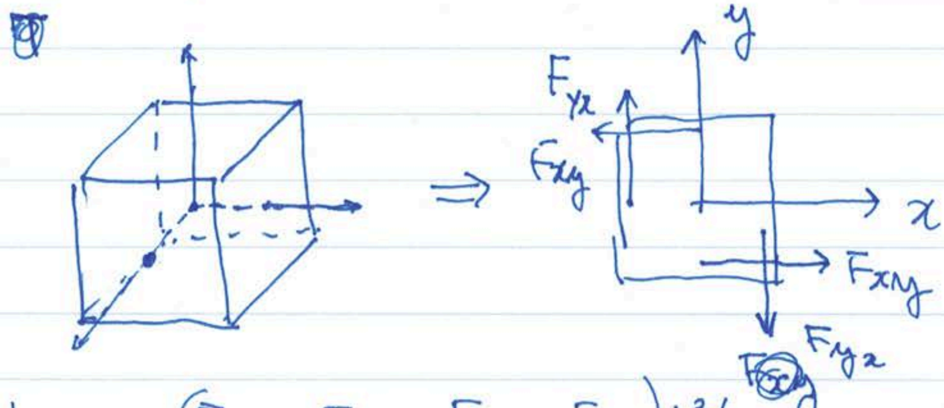
$A \cdot F_{xz}$: x-direction force felt by surface above

$A \cdot F_{yz}$: y- ....

$A \cdot F_{zz}$: z- ...

also. rate of momentum transfer.

C. symmetry



torque $(F_{xy} - F_{yx} + F_{xy} - F_{yx}) L^3/2$ $\Rightarrow$ $F_{xy} = F_{yx}$!!

moment of inertia $\sim \rho L^5$

D. pressure vs. stress (shear)

$$\|T\| = \underbrace{\begin{bmatrix} p & & \\ & p & \\ & & p \end{bmatrix}}_{\substack{\text{invariant under rotation} \\ \text{pressure} \to \text{pressure} \\ \text{spin } 0}} + \underbrace{\begin{bmatrix} & x & y \\ & & \cdot \\ & & \end{bmatrix}}_{\substack{\text{invariant under rotation} \\ \text{shear} \to \text{shear} \\ \text{spin } 2}}$$

symmetric trace-free

E: Maxwell stress

$$\mathbb{T} = \frac{1}{8\pi}\left[\left(\underline{E}^2 + \underline{B}^2\right) g - 2\left(\underline{E}\otimes\underline{E} + \underline{B}\otimes\underline{B}\right)\right]$$

F. Conservation of Momentum

$$\partial \underline{\Pi} / \partial t + \underline{\nabla} \cdot \mathbb{T} = 0$$

$\underline{\Pi}$: momentum density

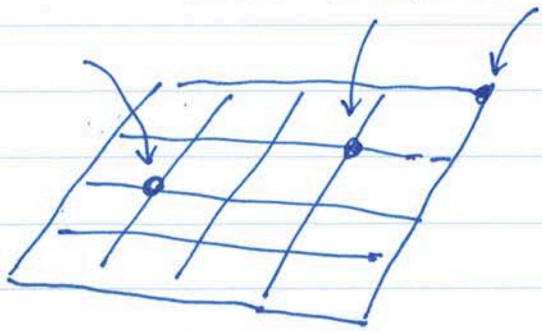
$\mathbb{T}$: also momentum flux

Special Relativity

A. Minkowski Spacetime.

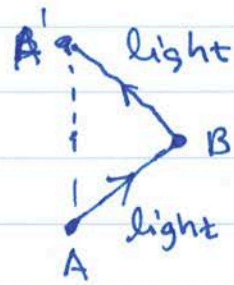
inertial reference frame.

ideal clock at each location



Orthogonal 3-D lattice, freely moving, non-rotating

clock syncronization



"Event" t: clock reading
x, y, z: location in lattice.

B. Principle of Relativity.

"Physics appears the same in every inertial frame"

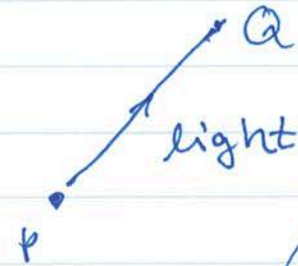
"Physical laws should be geometric, frame independent"

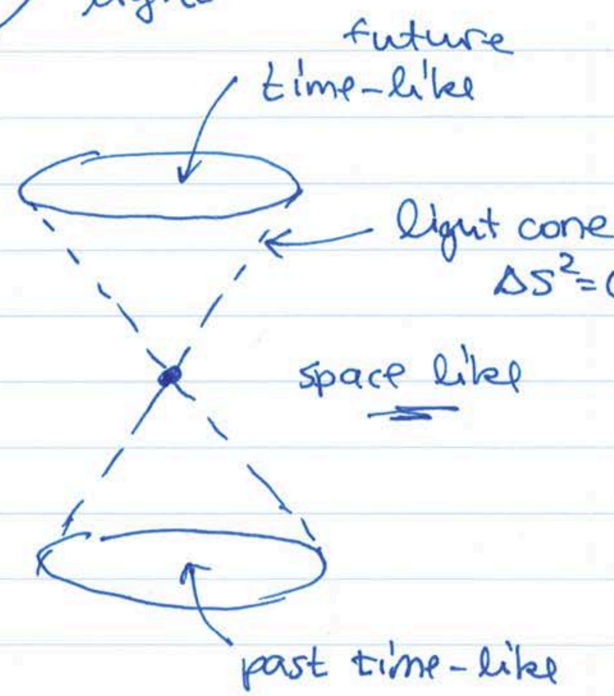
"Speed of light is the same for all inertial frames." ← Maxwell Equation.

C. · $\Delta S^2 \equiv -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \quad (c=1)$

in different inertial frames.

1. $\Delta S^2 = 0$ is preserved. sign of ΔS^2 then also preserved

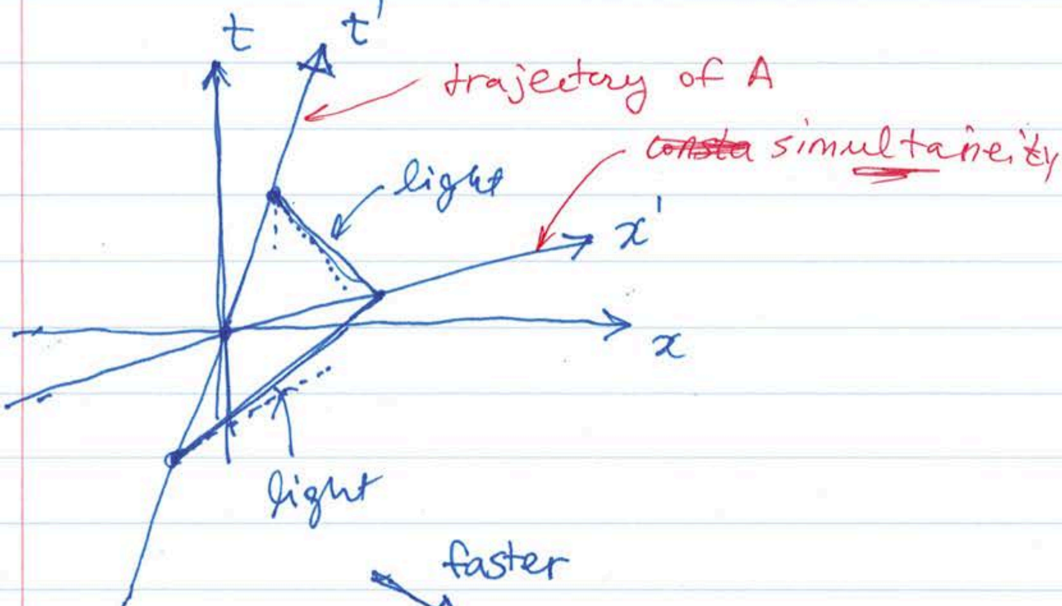


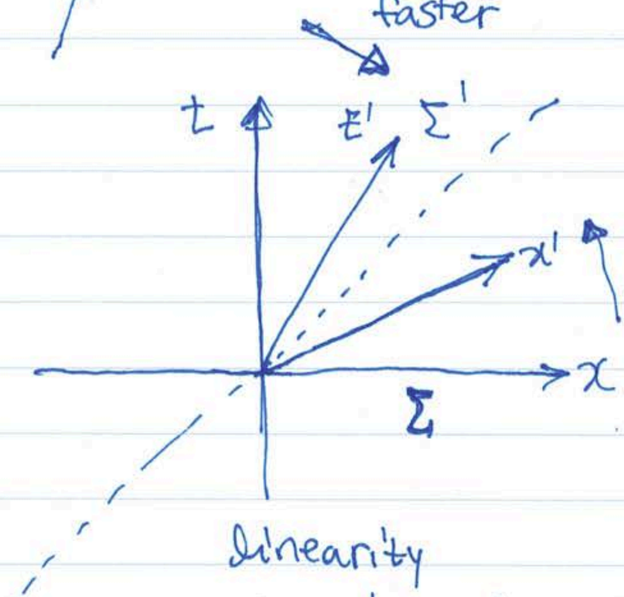


"simultaneity" is not preserved.

$\boxed{\text{Lorentz Transformation}}$

2. coordinate transformation is linear.





linearity
symmetry under exchange
preserving $\Delta S^2 = 0$

} ΔS^2 preserved

easy to show that "boost"

$$\begin{cases} t' = \gamma t + \gamma\beta x \\ x' = \gamma\beta x + \gamma t \end{cases} \qquad \gamma^2 = \frac{1}{1-\beta^2}$$

β : velocity of the Σ' frame wrt Σ

3. Metric

$$|\vec{A}|^2 = \Delta S^2 .$$

$$\vec{A}\cdot\vec{B} \equiv \left(|\vec{A}+\vec{B}|^2 - |\vec{A}-\vec{B}|^2\right)/4$$

$\vec{A}$: 4-vector. $\underline{A}$: 3-vector

4. orthonormal basis

$\vec{e}_\mu$: $\vec{e}_0, \vec{e}_1, \vec{e}_2, \vec{e}_3$ $\quad \vec{e}_\mu\cdot\vec{e}_\nu = \eta_{\mu\nu}$

$$\|\eta_{\mu\nu}\| = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \qquad g(\vec{e}_\mu, \vec{e}_\nu) = g_{\mu\nu} = \vec{e}_\mu\cdot\vec{e}_\nu$$

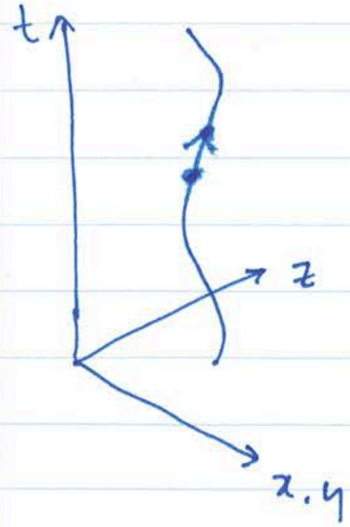
$\eta_{\mu\nu}$ in orthonormal basis.

~~Lorentz transforms~~

$\cancel{\vec{e}_\mu}$

D. Particle Kinetics.

1.



proper time

$$(\Delta\tau)^2 = +\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$
$$= -\Delta s^2$$

$$\boxed{\Delta\tau: \text{ time measured by clock carried by particle}}$$

use $\Delta\tau$ to parametrize the particle's trajectory.

$$\vec{u} = d\vec{x}/d\tau \qquad \vec{u}^2 = \frac{d\vec{x}\cdot d\vec{x}}{d\tau^2} = -1$$

4-momentum. $\vec{p} = m\vec{u}$.

$$\vec{p}^2 = -m^2$$

2. massless particle.

$m \to 0$. $d\tau \to 0$ with $d\zeta = d\tau/m$

then

$$\underset{\text{finite}}{\vec{p}} = \frac{\overset{\text{finite}}{d\vec{x}}}{\underset{\text{finite}}{d\zeta}} \qquad \vec{u} = \vec{p}/\underset{\text{infinite}}{m}$$

$$\vec{p}^2 = 0$$