Physics 136a, Week 1: Geometric Viewpoint

(Dated: September 28, 2011; due October 5, 2011)

The maximum number of points you can get for this assignment is 50, although you could choose to do problems that worth more than 50 points.

I. PROBLEMS

This week, we studied Chapter 1 and part of Chapter 2 (Secs. 2.1 – 2.4.1) of Blandford and Thorne. Problems 1 – 6 are relevant to Chapter 1 of Blandford and Thorne; Problems 7 and 8 are relevant to Chapter 2 of Blandford and Thorne. *Here I am referring to Version 1101.2.K of Chapter 1 and 1102.2.K of Chapter 2.*

1. **Tensor Basis.** [5 Points]

   If we view tensors as multi-linear functions of vectors, show that every tensor (e.g., rank 3) can be written in the following basis expansion:

   $$\mathbf{T} = T_{ijk}\mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \tag{1}$$

   Hint: try to show that when $T_{ijk} = \mathbf{T}(\mathbf{e}_i, \mathbf{e}_j, \mathbf{e}_k)$ is used, we indeed recover the tensor $\mathbf{T}$.

2. **The Slot Naming Index Notation** [10 Points]

   Exercise 1.5 of Blandford and Thorne

3. **Rotation in the x-y plane** [10 Points]

   Exercise 1.6 of Blandford and Thorne

4. **Cross Product and Curl** [15 Points]

   Exercise 1.7 of Blandford and Thorne

5. **Integral over a sphere** [15 Points]

   Exercise 1.10 of Blandford and Thorne

6. **Equations of motion for a perfect fluid** [15 Points]

   Exercise 1.12 of Blandford and Thorne

7. **Derivation of invariance interval** Δs^2 [15 Points]

   Yanbei used a different way of arriving at $\Delta s^2 = -t^2 + x^2 + y^2 + z^2$ than Blandford and Thorne. Read Section 2.2.2 for their derivation, and do exercise 2.2

8. **Another derivation of the invariance interval** Δs^2 [15 Points]

   (a) Show that a linear transformation $(t, x, y, z) \to (\bar{t}, \bar{x}, \bar{y}, \bar{z})$ must transform $\Delta s^2 = -t^2 + x^2 + y^2 + z^2$ into a quadratic form of $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ with the same signature $(-1, 1, 1, 1)$, which we shall denote as $\Delta \bar{s}^2$.

   (b) Explain why constant speed of light requires $\Delta \bar{s}^2 = 0$ on the cone of $-\bar{t}^2 + \bar{x}^2 + \bar{y}^2 + \bar{z}^2 = 0$.

   (c) Explain why $\Delta \bar{s}^2 = A$, with A a positive or negative constant must be a hyperboloid that asymptotes to the cone of $-\bar{t}^2 + \bar{x}^2 + \bar{y}^2 + \bar{z}^2 = 0$ when far enough away from the origin.

(d) Explain why that hyperbola must then be $-\bar{t}^2 + \bar{x}^2 + \bar{y}^2 + \bar{z}^2 = B$, with B a constant of the same sign as A.

(e) Argue that there in fact are enough points on the hyperboloid that the above can only happen if $\Delta\bar{s}^2 = (B/A)\Delta s^2$. Note that $B/A > 0$.

(f) Give a physical argument why B/A should be unity.