Physics 136b, Week 5: Vorticity

(Dated: due Feb 8, 2012)

The maximum number of points you can get for this assignment is **50**, although you could choose to do problems that worth more than 50 points.

There will be no lecture on Monday, Feb 6. However, you might want to use this time to view some fluid dynamics movies — actually seeing the flow patterns will be very helpful for building intuition. Here is a link to movies made by the National Committee for Fluid Mechanics Films (NCFMF):

`http://web.mit.edu/hml/ncfmf.html`

Of particular interest to this week's discussions is "Fundamentals-Boundary Layers", which explains laminar boundary layers and how they transition into turbulence. Also interesting are "Vorticity, Part 1", "Vorticity, Part 2", which cover the basics of vorticity, Kelvin Circulation Theorem, etc., and "Low Reynolds Number Flow". All these movies add together to roughly one and a half hours.

This week we discussed elastodynamics, covering 13.8 and 14.1 – 14.4 of Blandford and Thorne (BT). [See version version 1113.1.K of Chapter 13 and 1114.1.K of Chapter 14.]

1. **Relativistic Bernoulli Theorem.** Exercise 13.18 of BT. [15 points]
2. **Vorticity and incompressibility.** Exercise 14.1 of BT. [15 Points]
3. **Joukowski's Theorem.** Exercise 14.4 of BT. [15 Points]
4. **Stokes Flow: another application of vector Harmonics.** [15 Points]
   (a) Show that the Stokes' Flow satisfies
   $$\vec{\nabla} \times (\vec{\nabla} \times (\vec{\nabla} \times \vec{v})) = 0\,, \quad \vec{\nabla} \cdot \vec{v} = 0\,. \tag{1}$$
   (b) Show that if we use
   $$l(l+1)\frac{u(r)}{r}\vec{Y}^{lm} + \left[\partial_r u(r) + \frac{u(r)}{r}\right]\vec{\Psi}^{lm} \tag{2}$$
   then $\vec{\nabla} \cdot \vec{v} = 0$ is already satisfied. Show that the boundary condition we have at infinity can be satisfied using only an $l = 1$ component.
   (c) Focus on $l = 1$, derive the differential equation to be satisfied by u (perhaps with the help of mathematica). You will find 4 homogeneous solutions, each of the form r^α, i.e., some power of r.
   (d) Determine the solution using boundary condition at $r = a$ and $r = +\infty$.
5. **Stationary Laminar Flow down a Long Pipe.** Exercise 14.11 of BT. [15 Points]