Physics 136a, Week 3: Kinetic Theory & Statistical Mechanics

(Dated: October 20, 2011; due **Wednesday October 26, 2011**)

The maximum number of points you can get for this assignment is **50**, although you could choose to do problems that worth more than 50 points.

This week, we kinetic theory and statistical mechanics. This corresponds to Secs. 3.6 – 3.7 (Version 1103.1) and Secs. 4.1 - 4.4.2, 4.5 and 4.6 (Version 1104.3) of Blandford and Thorne (BT).

1. BT Exercise 3.14 [15 Points]
2. BT Exercise 3.15 [15 Points]
3. BT Exercise 3.20 [20 Points]
4. Derivation of Canonical Ensemble's Distribution Function [15 Points]

   In class, we argued that $\log\rho = -\beta\mathcal{E}$ because $\rho = \Pi_a \rho_a$ and $\mathcal{E} = \sum_a \mathcal{E}_a$. Let us prove it here mathematically.

   (a) Suppose we have three functions f, g, and h, so that $\forall x, y$,

   $$f(x)g(y) = h(x+y) \tag{1}$$

   By taking partial derivative with respect to x and y, argue that for all x and y, we must have

   $$f'(x)/f(x) = g'(y)/g(y) \tag{2}$$

   Show that this means we have a constant β, such that

   $$f(x) = Ae^{-\beta x}, \quad g(y) = Ae^{-\beta y} \tag{3}$$

   (b) Use part (a) to prove that $\rho = Ce^{-\beta\mathcal{E}}$.