Physics 136a, Week 6: Random Processes and Geometric Optics

(Dated: November 10, 2011; due **Wednesday November 16, 2011**)

The maximum number of points you can get for this assignment is **50**, although you could choose to do problems that worth more than 50 points.

This week we discussed fluctuations and geometric optics. This corresponds to Sections 6.4 – 6.8 (Version 1106.5) and 7.1 – 7.3.4 (Version 1107.2) of Blandford and Thorne (BT).

1. **Spectral Density** [15 Points]

   Show that the spectral density of a Gaussian random process, $x(t)$ defined over the time interval of $(-T/2, +T/2)$,

   $$S_x^T(f) = \frac{2}{T}\left(\int_{-T/2}^{+T/2} x(t_1)e^{2\pi i f t_1}dt_1\right)\left(\int_{-T/2}^{+T/2} x(t_2)e^{-2\pi i f t_2}dt_2\right) \tag{1}$$

   converges to a non-random variable with unit probability, as $T \to +\infty$.

   [Hint: this basically requires us to show that $\langle (S_x^T)^2\rangle - \langle (S_x^T)\rangle^2 \to 0$. If we expand the expression of $\langle (S_{xy}^T)^2\rangle$, we find out that it contains an integral with integrand equal to $\langle x(t_1)x^*(t_2)x(t_1')x^*(t_2')\rangle$. First use Gaussianity and show that,

   $$\begin{aligned}\langle x(t_1)x(t_2)x(t_1')x(t_2')\rangle &= \langle x(t_1)x(t_2)\rangle\langle x(t_1')x(t_2')\rangle + \langle x(t_1)x(t_1')\rangle\langle x(t_2)x(t_2')\rangle \\ &+ \langle x(t_1)x(t_2')\rangle\langle x(t_2)x(t_1')\rangle\end{aligned} \tag{2}$$

   Then show that the latter two terms give contributions $\sim 1/T$.]

2. **Cosmological Density Fluctuations.** Exercise 6.6 of BT [15 Pionts]

3. **Optimal Filtering.** Exercise 6.10 of BT [15 Points]

4. **Generalized Fluctuation-Dissipation Theorem.** Exercise 6.16 of BT [15 Points]

5. **Gaussian Wave Packet.** Exercise 7.2 of BT [15 Points]

6. **Conservation law along the ray.** Exercise 7.3 of BT [15 Points]