# Ph136 Solution 10

11.1 $\Theta = S_{ii} \qquad R_{ij} = \frac{1}{2}(S_{ij} - S_{ji})$

$$\Sigma_{ij} = \frac{1}{2}(S_{ij} + S_{ji}) - \frac{1}{3} g_{ij} S_{kk}$$

$$\Rightarrow \frac{1}{3}\Theta \delta_{ij} + \Sigma_{ij} + R_{ij} = S_{ij}$$

11.4 $(K + \frac{1}{3}H)\nabla(\nabla\cdot\vec{\xi}) + H\nabla^2\vec{\xi} + \rho\vec{g} = 0 \qquad ①$

Take divergence and then take gradient

notice that $\nabla(\nabla\cdot(\nabla(\nabla\cdot\vec{\xi}))) = \nabla(\nabla\cdot(\nabla^2\vec{\xi})) = \partial_k\partial_i\partial_j^2\xi_i$

$$\Rightarrow \nabla^2(\nabla(\nabla\cdot\vec{\xi})) = 0$$

On the other hand, taking $\nabla^2$ of ① $\Rightarrow \nabla^2\nabla^2\vec{\xi} = 0$

Also $\Rightarrow \nabla^2\Theta = \nabla^2(\nabla\cdot\vec{\xi}) = -\frac{H}{K+\frac{1}{3}H}\nabla^2(\nabla\cdot\vec{\xi})$

$$\Rightarrow \nabla^2\Theta = 0$$

11.6 $U = \frac{1}{2}\xi_{i;j} Y_{ijkl}\xi_{k;l}$

$$\mathcal{E} = \int_V U\,dV = \int \frac{1}{2}\xi_{i;j} Y_{ijkl}\xi_{k;l}$$

$$\delta\mathcal{E} = \int_V \delta\xi_{i;j} Y_{ijkl}\xi_{k;l}\,dV = +\underbrace{\int_{\partial V}\delta\xi_i Y_{ijkl}\xi_{k;l}\,d\Sigma_j}_{\text{surface}} - \underbrace{\int_V \delta\xi_i (Y_{ijkl}\xi_{k;l})_{;j}\,dV}_{\text{bulk}}$$

$$\Rightarrow T_{ij} = -Y_{ijkl}\xi_{k;l}$$