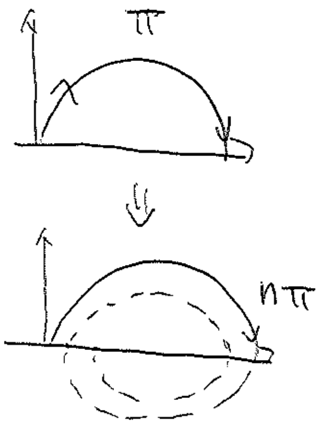
Ph136 Solution 8

8.9 (a) This is trivial because at $f_n = \frac{f}{n}$, each original

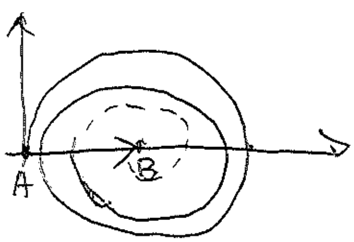


turns to $n\pi$ rotations in the phase diagram

So if n is even, they cancel with each other and It's a dark point

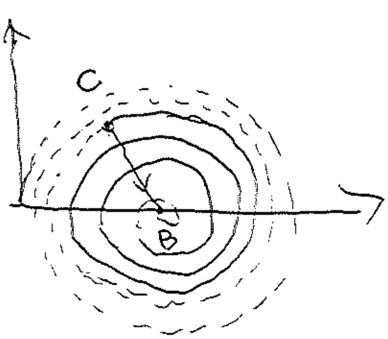
if n is odd, it's bright and hence a focus point.

(b) If there were no disk, the diagram is like



$$|\psi| = |\psi_{AB}|$$

If the disk is present, the digram is



$$|\psi'| = |\psi_{CB}| \approx |\psi_{AB}|$$

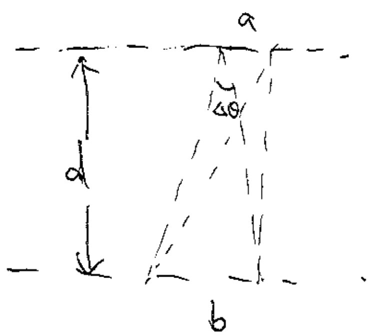
So the brightness is roughly the same as if there were no disk.

Solution 8

8.10 (a) $\Delta k \cdot a \sim \Delta\phi$ and $\Delta\theta \sim \frac{\Delta k}{k}$

$$\Rightarrow \Delta\theta \sim \frac{\Delta\phi}{ak} \sim \frac{\lambda}{a}\Delta\phi$$

(b) Apparently $\Delta\theta \sim \frac{b}{d}$



$$\Rightarrow \lambda d \sim ab\Delta\phi \sim ab$$

So Fresnel length $\sqrt{\lambda d} \sim \sqrt{ab}$

$$a \sim \frac{u}{f} \sim \frac{30\,m/s}{100/s} \sim 0.3\,m$$

$$\Delta\phi \sim \frac{a}{\lambda}\Delta\theta \sim \frac{0.3}{0.5\times10^{-6}}\,1\text{ arc second} \sim \frac{0.3}{0.5}\,4.8$$

$$\sim 3$$

It's higher than 1 but nevertheless same order of magnitude

Solution 8

8.15 (a) The phase difference due to irregularity is $e^{-2ikh(x)}$

$$\Rightarrow \psi'(x) = \psi^{G}(x)\exp[-i2kh(x)] \approx \psi^{G}(x)(1-i2kh(x))$$

The factor 1 is the coherent reflected beam, So the scattered light:

$$\psi^{S}(x) = -i\psi^{G}(x)\,2kh(x)$$

(b) $$\psi^{S}(\theta) \propto \int \psi^{G}(x)\,kh(x)\,e^{ikx\cdot\theta}\,d\Sigma$$

$$\propto FT[\psi^{G}(x)h(x)]$$

For angle $\theta \Rightarrow k \sim \frac{2\pi\theta}{\lambda} \Rightarrow \lambda_{mirror} \sim \lambda/\theta$

For LIGO, $L = 4km$, $R = 60cm$, consider scattering at $L/2$

$$\Rightarrow \theta \sim \frac{2R}{L} \sim 3\times10^{-4}$$

$$r_F = \sqrt{\lambda L/2} \sim 4\times10^{-2}m$$

So $\lambda_{mirror} \sim \lambda/\theta \sim 0.3\times10^{-2}m < r_F$

So they are in the Fraunhofer hegiom.

Solution 8

8.15 (C) The isotropically scattered component has

$\lambda_m \sim \lambda \simeq 1\,\mu m$

(d)(e)

$$P_{10} \sim \frac{-ik}{2\pi\ell} e^{ik\ell} \exp\left(\frac{ikR^2(\phi)}{2\ell}\right)$$

$$P_{21} \sim \frac{-ik}{2\pi(L-\ell)} e^{ik(L-\ell)} \exp\left(\frac{ikR^2(\phi)}{2(L-\ell)}\right)$$

$$\Rightarrow P(\phi) \propto P_{10}P_{21}d\phi \propto \exp\left(\frac{ikR^2(\phi)}{2}\left(\frac{1}{\ell}+\frac{1}{L-\ell}\right)\right)d\phi$$

The time variation of the baffle vibration

$$\rightarrow P(\phi,t) = e^{ik\frac{\delta R^2(\phi,t)}{2\ell_{red}}}\,\bar{P}(\phi)$$

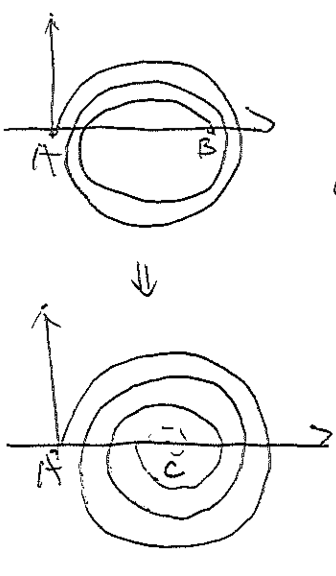
Moreover, since $R(\phi)$ is randomly distributed, the mean part

$\Psi_{noise} \sim \int d\phi\, \bar{P}(\phi)\, \Psi_s$ is greatly reduced

6 Area of Fresnel zone $\sim \pi\lambda L/2 \times 6 \sim 2\pi R\,\Delta h$

$\Rightarrow \Delta h \sim 1\,cm$

The coherent superposition may lead to $\Psi \sim |AB|$



But the randomized end point in the phase diagram $\Rightarrow \Psi' \sim |AC| \sim \frac{1}{2}|AB|$

$$\Rightarrow \frac{I'}{I} \sim 1/4$$